

# Quantum Uncertainties in the Schmidt Basis Given by Decoherence

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## Abstract

A common misconception is that decoherence gives the eigenstates that we observe to be fairly definite about a subsystem (e.g., approximate eigenstates of position) as the elements of the Schmidt basis in which the density matrix of the subsystem is diagonal. Here I show that in simple examples of linear systems with gaussian states, the Schmidt basis states have as much mean uncertainty about position as the full density matrix with its combination of different possibilities.

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A question puzzling some of us scientists is why our observations are fairly definite, and why what appears to us visually to be fairly definite is usually something like approximate positions of objects that we see. One might say that what we see is presumably determined by the firings of neurons in the retinas at the backs of our eyes, so that a particular pattern of firings gives a visual impression of the locations of objects that we see. Then the question is why we are visually aware of a fairly definite pattern of retinal neuron firings. One might go on to say that this is because a fairly definite pattern of these retinal neuron firings induces a fairly definite pattern of neuron firings or some other property in some more central part of the brain where the visual awareness may be postulated to occur, but this just pushes the question back to why we are aware of those brain properties, rather than of superpositions of them. If we assume that for particular brain properties, there are corresponding visual awarenesses of objects that appear to have fairly definite positions, rather than of combinations of different positions, the question is then what is the preferred basis of states for the subsystem of these brain properties, such that each basis state leads mainly to a single definite visual awareness.

The mystery arises because in quantum theory, unitary evolution would almost always lead to a state of the brain subsystem that is a mixed state of the particular brain properties that each lead to fairly definite visual awarenesses. If the brain state in quantum theory is a mixed state of many brain properties, what picks out the particular brain states that each lead mainly to a fairly definite observation that one is aware of having? Or, if we assume that the process of vision maps the relative positional configuration of an observed object to a corresponding brain property, what picks out these particular states of the object (rather than superpositions of them) that each lead to a fairly definite visual awareness of the object? Observationally, these seem to be approximate position eigenstates of the object, but why is that true?

Traditionally it was postulated that the quantum state (of a closed system) not only has the unitary evolution of the Schrödinger equation, but that at certain times the unitary evolution is broken by the so-called “collapse of the wavefunction” to return it to a macroscopically definite quantum state [1], such as an approximate position eigenstate of observed objects. This collapse was supposed to occur during measurements, but usually it was left rather vague what precisely constituted a measurement and exactly when the collapse of the wavefunction would occur.

More recently it has become widely recognized that quantum subsystems of the universe rapidly become entangled with their environments through generic interaction processes called decoherence, so that the subsystems are not in pure states but mixed states, described by density matrices or density operators that are not the unit-rank projection operators that are the density operators of pure quantum states [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34]. It is sometimes assumed that it is the eigenstates of the density operator of a subsystem produced by decoherence (see, e.g., [3, 9, 11], who use these eigenstates) that are the particular states of a subsystem that each lead to fairly definite visual awarenesses. For example, I got that impression from some recent statements of Raphael Bousso and Leonard Susskind [35], in a paper that has several other interesting ideas whose truth or falsehood seems to be rather independent of how I interpreted their statements about decoherence. They wrote, “Decoherence explains why observers do not experience superpositions of macroscopically distinct quantum states, such as a superposition of an alive and a dead cat. . . . Decoherence explains the ‘collapse of the wave function’ of the Copenhagen interpretation as the non-unitary evolution from a pure to a mixed state, resulting from ignorance about an entangled subsystem  $E$ . It also explains the very special quantum states of macroscopic objects we experience, as the elements of the basis in which the density matrix  $\rho_{SA}$  is diagonal.”

Here I wish to correct the misconception that seemed to me to be implicit in the last sentence above, that the eigenstates of the subsystem density matrix (the Schmidt basis for it) each have the variables that are observed to be macroscopically definite (e.g., positions) in macroscopically definite states. It is not generically true that the Schmidt basis, in which the subsystem density matrix is diagonal, would have basis states (the eigenstates of the subsystem density matrix) in each of which there is a single macroscopic state, such as the approximate position eigenstates that we appear to observe in each individual visual observation. It may be true that with interactions that are local in space, the density matrix in a basis that each has an appropriate single macroscopic state (e.g., an appropriate superposition of quantum microstates that each have the same unique macroscopic values) is often approximately diagonal, but the basis in which the density matrix really is precisely diagonal is, as I shall show for a wide class of simple examples, far from each having definite macroscopic states. In particular, I shall show that for many simple examples the mean uncertainty of the position variables in each of the Schmidt basis states is just as great as the full uncertainty that they have in the complete quantum density matrix of the subsystem.

To quantify how much mean uncertainty there is in a certain operator (e.g., for a macroscopic variable such as center-of-mass position) in a certain basis (e.g., the Schmidt basis), I shall define a dimensionless *Mean Observable-Basis Uncertainty* (MOBU)  $U_{OB}$  as the ratio of the mean variance of the observable  $O$  (a hermitian operator) in a basis  $B = |i\rangle$  ( $i = 1 \dots m$ ) of  $m$  pure states for a quantum subsystem of Hilbert-space dimension  $n$  and with a mixed state given by the density operator  $\rho$ , to the full variance of the observable in the same mixed state. If  $B$  is the Schmidt basis, one has that  $m = n$  and that the  $|i\rangle$  are the orthonormal eigenvectors of the density operator, which can be written as  $\rho = \sum_{j=1}^n p_j |j\rangle\langle j|$  with nonnegative eigenvalues  $p_j$  that sum to unity (and which are often interpreted to be the proba-

bilities that the quantum subsystem is in each of the  $n$  pure states  $|j\rangle$  that are the orthonormal eigenvectors of the density operator). However, I shall give a general definition of the MOBU  $U_{OB}$  for an arbitrary basis  $B$ , without even assuming that the  $m$   $|i\rangle$  are orthonormal.

The full variance of the observable  $O$  in the (normalized) density matrix of the quantum subsystem is  $(\Delta O)^2 \equiv \langle (O - \langle O \rangle)^2 \rangle = \langle O^2 \rangle - \langle O \rangle^2 \equiv \text{tr}(\rho O^2) - [\text{tr}(\rho O)]^2$ . The variance in the pure state  $|i\rangle$  (not necessarily assumed to be normalized) is  $(\Delta O_i)^2 \equiv \langle i|O^2|i\rangle/\langle i|i\rangle - (\langle i|O|i\rangle/\langle i|i\rangle)^2$ . Define the probabilities  $P_i$  for the basis states  $|i\rangle$ , given the subsystem density operator  $\rho$ , to be  $P_i \equiv r_i/N$  with  $r_i \equiv \langle i|\rho|i\rangle/\langle i|i\rangle$  being the relative probability for the basis state  $|i\rangle$  and with  $N \equiv \sum_{i=1}^m r_i$  being the normalization factor, which will be unity if  $m = n$  and if the  $|i\rangle$  are orthogonal. For the Schmidt basis  $|j\rangle$  of  $m = n$  orthonormal eigenvectors of the density operator  $\rho$ ,  $P_j = r_j = \langle j|\rho|j\rangle = p_j$ , but for a generic basis I shall reserve  $p_j$  for the  $n$  eigenvalues of  $\rho$  and use  $P_i$  for the probabilities of the  $m$  basis states  $|i\rangle$ . Then the subsystem has a mean variance of the observable  $O$  in the basis  $B = |i\rangle$  that can be defined to be  $(\Delta O_B)^2 \equiv \sum_{i=1}^m P_i (\Delta O_i)^2$ . Finally, define the dimensionless *Mean Observable-Basis Uncertainty* (MOBU) as  $U_{OB} \equiv (\Delta O_B)^2/(\Delta O)^2$ , the ratio of the mean variance of the observable  $O$  in the basis  $B$  to the full variance of  $O$ .

Given an observable  $O$  that represents what is believed to be observed to have definite values (e.g., macroscopic positions), a goal would be to find a basis of states  $B = |i\rangle$  that gives a small value of the Mean Observable-Basis Uncertainty  $U_{OB}$ . Of course, one can just choose an orthonormal basis of eigenvectors of  $O$ , and then in each pure eigenstate  $|i\rangle$  of  $O$ ,  $(\Delta O_i)^2 = 0$ , so the mean variance of the observable in this basis is  $(\Delta O_B)^2 = 0$ , giving  $U_{OB} = 0$  if the full variance  $(\Delta O)^2$  of  $O$  in the density operator of the quantum subsystem is positive. However, the question here is whether one gets a small  $U_{OB}$  from the Schmidt basis of eigenvectors of the subsystem density operator  $\rho$ .

One can easily see that if there is no restriction on the basis (even if it is required to be an orthonormal basis of the same dimension  $n$  as the Hilbert space of the subsystem under consideration), then the MOBU can be any positive number (including infinity, if the observable has no full variance in the density operator of the subsystem). However, it is restricted to be no greater than unity for the Schmidt basis  $|i\rangle = |j\rangle$ , as I shall now show.

Assuming that  $B = |i\rangle$  is the Schmidt basis of  $m = n$  orthonormal eigenstates of the density operator, which can then be written as  $\rho = \sum_{i=1}^m P_i |i\rangle\langle i|$  with non-negative eigenvalues  $P_i = p_i$  that are the same as what was defined above to be the probabilities for these particular basis states, define the mean value of the observable  $O$  in the pure state  $|i\rangle$  to be  $O_i \equiv \langle i|O|i\rangle$  and the mean value in the full mixed state to be  $O_m \equiv \langle O \rangle \equiv \text{tr}(\rho O) = \sum_{i=1}^n p_i \langle i|O|i\rangle = \sum_{i=1}^m P_i O_i$ . Interpret  $O_i$  and  $O_m$  to mean these expectation values multiplied by the identity operator when they are used inside quantum inner products. Then the variance of the observable  $O$  in the normalized pure state  $|i\rangle$  is  $(\Delta O_i)^2 = \langle i|(O - O_i)^2|i\rangle$ , the mean variance of  $O$  in the Schmidt basis  $B$  is  $(\Delta O_B)^2 = \sum_{i=1}^m P_i \langle i|(O - O_i)^2|i\rangle$ , and the full variance of  $O$  in the mixed state of the subsystem is  $(\Delta O)^2 = \sum_{i=1}^m P_i \langle i|(O - O_m)^2|i\rangle$ , which is greater than the mean variance by the nonnegative excess  $E = (\Delta O)^2 - (\Delta O_B)^2 = \sum_{i=1}^m P_i \langle i|[(O - O_m)^2 - (O - O_i)^2]|i\rangle = \sum_{i=1}^m P_i \langle i|(O_i - O_m)(2O - O_i - O_m)|i\rangle = \sum_{i=1}^m P_i (O_i - O_m)^2 \geq 0$ . Therefore, the Mean Observable-Basis Uncertainty is  $U_{OB} \equiv (\Delta O_B)^2/(\Delta O)^2 = 1 - E/(\Delta O)^2 \leq 1$  for the Schmidt basis.

Thus the Schmidt basis never gives a mean variance of an observable  $O$  in its basis states that is larger than the full variance of  $O$ , unlike what is possible with other bases. However, it can give a mean variance as large as the full variance, and hence a MOBU value of  $U_{OB} = 1$ , if the mean value of  $O$  in each Schmidt basis state is the same,  $O_i = O_m$  for all  $i$  for which  $P_i > 0$ . Next I shall show that this is what indeed occurs for the position observable in simple models of linear coupling of

harmonic oscillators in which the subsystem density matrix has a gaussian form as the exponential of a negative quadratic expression in the positions and/or momenta.

There has been an extensive study of quantum models in which a free particle or harmonic oscillator (which I shall take as the quantum subsystem of interest) interacts linearly with a collection of other harmonic oscillators (which I shall call the environment; in some cases it may be taken to be a heat bath) [36, 37, 38, 39, 40, 41, 42]. For simplicity, one often takes the initial density matrix for the whole coupled system to have a gaussian form, such as a product (no initial entanglement) of a pure or mixed gaussian state for the subsystem of interest and another pure or mixed state for the environment (such as a thermal state). The linear coupling leads to entanglement between the subsystem and the environment, but for free particles or harmonic oscillators with linear couplings between the positions and/or momenta, the full quantum state retains its gaussian form (proportional to the exponential of a negative quadratic expression in all the positions and/or momenta). It is then easy to see that tracing over the environment gives a gaussian density matrix for the subsystem of interest [43].

By performing a canonical transformation of the position and momentum operators (which generically includes not only rotations in the phase space but also shifting the expectation values for the transformed positions and momenta to zero and rescaling the positions and momenta by a squeeze), one can write the gaussian density matrix as a product of thermal gaussian states for each degree of freedom for the transformed system, with thermality defined with respect to a ‘Hamiltonian’ that for each transformed degree of freedom is half the sum of the squares of the transformed position and momentum variables [42]. The Schmidt basis of eigenstates of this subsystem density matrix are then the products of the energy eigenstates of this ‘Hamiltonian’ for each degree of freedom. Each of these eigenstates has zero expectation values for each of the transformed position and momentum variables. Hence,

it has the same values (though generically not zero) for the expectation values of the original position and momentum variables. Therefore, for any observable  $O$  that is a linear combination of position and momentum variables, the Schmidt basis for such a gaussian state of the subsystem gives the same value for the expectation value  $O_i \equiv \langle i|O|i\rangle$  for each Schmidt basis state,  $O_i = O_m = \langle O \rangle = \text{tr}(\rho O)$ . By the result above, this leads to the excess of the full variance of  $O$  over the mean variance of  $O$  in the Schmidt basis being  $E = 0$ , so the dimensionless Mean Observable-Basis Uncertainty is  $U_{OB} = 1$ .

Thus, on average, each eigenstate of the density operator for the quantum subsystem gives no less uncertainty for a position (or momentum, or linear combination of position and momentum) observable than the full density matrix does. That is, each element of the Schmidt basis given by decoherence leads to a mean uncertainty in position just as great as the uncertainty given by the entire density matrix of the subsystem. For these common examples of linearly coupled harmonic oscillators in gaussian states, decoherence does not lead to eigenstates of the subsystem density operator that are sharp in position.

It would be interesting to analyze quantum systems that are not in gaussian states, either from imposing nongaussian initial states or from having nonlinear couplings, to see how the Mean Observable-Basis Uncertainty behaves. For example, if one starts with the subsystem having a superposition of two widely separated gaussian states but continues to restrict to linear couplings for simplicity, does the MOBU evolve to become very small, or do the eigenstates of the density matrix for the subsystem each continue to have significant contributions from the two widely separated locations to keep the MOBU of the order of unity? Such considerations will be left for future research.

It would also be interesting to calculate the MOBU for the more sophisticated ‘pointer bases’ that have been proposed [4, 5, 7, 8, 10, 23, 24, 28, 30, 44, 45, 46, 47].



Although none of these have been precisely defined for a generic situation, they are variously described as “the eigenvectors of the operator which commutes with the apparatus-environment interaction Hamiltonian” and “contains a reliable record of the state of the system” [4], the eigenstates of the observable of the apparatus “which is most reliably recorded by the environment” [5], the states “which become minimally entangled with the environment in the course of the evolution” [24], “the pure states least affected by decoherence” [28], “states that produce the least entropy,” “states that are the easiest to find out from the imprint they leave on the environment,” “states that can be deduced from measurements on the smallest fraction of the environment,” and as “states for which it takes the longest to lose a set fraction their initial purity” [45]. It is also admitted that “There is no *a priori* reason to expect that all of these criteria will lead to identical sets of preferred states,” though it is “reasonable to hope that, in the macroscopic limit in which classicality is indeed expected, differences between various sieves should be negligible” [28]. I do suspect that often the MOBU for the position observable would be rather small for pointer bases that are suitably defined, but that remains to be calculated.

Because of the ambiguity of which of the many qualitative criteria to choose for pointer bases, and of how to make any of them precise, I do not think these many different ideas about pointer bases are the final answer to the question of how to explain our observations, though they do seem to be important steps in the right direction. To me it appears simplest to postulate that measures or probabilities for our observations are given by the expectation values of certain definite ‘awareness operators,’ but we do not yet know what they are and how the contents of our observations may correlate with these operators themselves [48, 49, 50].

In conclusion, each fairly definite location that we observe visually for an object does not appear to have the form given by any of the eigenstates of the subsystem density operator after decoherence, at least for linearly interacting systems with

gaussian density operators. So this simple-minded idea from decoherence (already criticized in [10, 23, 30]) seems somewhat incoherent.

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